ON THE ELECTRIC DIPOLE POLARIZABILITY
OF THE THREE-HADRON BOUND SYSTEM

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A simple analytical expression for the electric dipole polarizability of the three-hadron bound system having only one stable bound state has been derived neglecting by the higher orbital components of the off-shell three-body transition matrix at the energy of the bound state. As a case in point, we have estimated the electric dipole polarizability of the triton, using a cluster triton wave function and the Hulthén potential to describe the related $p-n$ and $n-d$ bound states.

Keywords: Electric dipole polarizability; triton.

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1. Introduction

Data on the electric polarizabilities of the lightest nuclei α_E contain a valuable information on the nuclear force between nucleons.

For the deuteron, the currently available values of $\alpha_E(^2\mathrm{H})$ obtained by the direct measurement of deviation from Rutherford scattering of the deuteron on a heavy nucleus below the Coulomb barrier[1] and extracted from photoabsorption data[2],

$$\alpha_E(^2\mathrm{H}) = 0.70 \pm 0.05 \ \mathrm{fm}^3 \ (\text{Ref. 1}) \ \text{ and } \ \alpha_E(^2\mathrm{H}) = 0.61 \pm 0.04 \ \mathrm{fm}^3 \ (\text{Ref. 2}), \quad (1)$$

are a little distinguished between themselves.

For the nucleus $^3\mathrm{He}$, contrastingly, the corresponding values of the polarizability, obtained by the direct way[3] and from experimental photoabsorption data[4],

$$\alpha_E(^3\mathrm{He}) = 0.25 \pm 0.04 \ \mathrm{fm}^3 \ (\text{Ref. 3}) \ \text{ and } \ \alpha_E(^3\mathrm{He}) = 0.130 \pm 0.013 \ \mathrm{fm}^3 \ (\text{Ref. 4}), \quad (2)$$

are distinguished by a factor of two.

For the nucleus $^3\mathrm{H}$ no measurement of the electric dipole polarizability has been performed to the present time.

The polarizability of the nucleus $^4\mathrm{He}$ was experimentally found[4−6] to be less by about an order of magnitude than that of the deuteron.

Calculations of the deuteron electric polarizability, carried out with the realistic nucleon-nucleon interaction potentials, lead to the values of $\alpha_E(^2\mathrm{H}) = 0.6328(17) \ \mathrm{fm}^3 \ (\text{Ref.7})$, being closer to the data of Ref. 2 in (1).

Furthermore, examining theoretically the anisotropy of the deuteron deformation in the electric field caused by the tensor character of the $n-p$ interaction, the separate components of the deuteron electric polarizability $\alpha_E^{|M|}$, the longitudinal component (with the deuteron spin along the electric field) α_E^1 and the transverse

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one α_E^0, have been calculated in Ref. 8. (The above electric polarizability $\alpha_E(^2\mathrm{H})$ is the averaged value of the components $\alpha_E^{|M|}$, $\alpha_E(^2\mathrm{H}) = \frac{2}{3}\alpha_E^1 + \frac{1}{3}\alpha_E^0$.)

Computations of the scalar and tensor deuteron polarizabilities have also been performed in the framework of the effective field theory that uses space-time and global chiral symmetries of the quantum chromodynamics consistently describing pion propagation and relativistic effects[9−11]. The results for the electric deuteron polarizabilities obtained in the cited works in the leading and next-to-leading orders agree with the values calculated in the traditional nuclear physics with the application of the potential models.

The results of calculations of the $^3\mathrm{He}$ polarizability, $\alpha_E(^3\mathrm{He}) = 0.145$ fm^3 (Ref. 12), $\alpha_E(^3\mathrm{He}) = 0.153(15)$ fm^3 (Ref. 6) and $\alpha_E(^3\mathrm{He}) = 0.149(5)$ fm^3 (Ref. 13), support the experimental result in (2) that has been obtained from the photoabsorption data (Ref. 4). Also, the results of calculations of the triton polarizability, $\alpha_E(^3\mathrm{H}) = 0.139(2)$ fm^3 (Refs. 13 and 14), turned out to be close to those of the $^3\mathrm{He}$ polarizability[6,12,13]. (A little larger value of $\alpha_E(^3\mathrm{He})$ may be assigned to the repulsive Coulomb interaction between the two protons in the nucleus $^3\mathrm{He}$ causing the charge symmetry violation.)

In the previous paper[15], we have predicted the value of the electric dipole polarizability of the only three-body lambda hypernucleus — the lambda hypertriton $^3_\Lambda\mathrm{H}$. It was found that $\alpha_E(^3_\Lambda\mathrm{H})$ is close to 3 fm^3 exceeding the polarizability of the ordinary three-body nuclei by an order of magnitude and even the recently measured polarizability of the nucleus $^6\mathrm{He}$, $\alpha_E(^6\mathrm{He}) = 1.99(40)$ fm^3 (Ref. 6).

This paper is devoted to development of a method of direct determination of the electric dipole polarizability of the three-particle bound system, leaning upon solution of the three-body problem at the negative bound-state energy without necessity of finding the continuum wave functions. In Sec. 2, a general formalism for determining the polarizability of the three-hadron complex is worked out. The electric polarizability of the three-hadron nucleus is expressed in terms of partial derivatives of the bound-state wave function in momentum space and selected higher partial components of the three-body off-shell transition matrix. In Sec. 3, the developed formalism is applied in the case of a simple physically justified (cluster) model of the wave function of the triton (a bound complex of one proton and two neutrons). Neglecting higher partial components of the transition matrix we obtain a simple formula for estimation of the triton electric dipole polarizability. The results of corresponding calculations of $\alpha_E(^3\mathrm{H})$ based on the known low-energy data for the $p-n$ and $d-n$ interactions are described in Sec. 4. Conclusions are drawn in Sec. 5.

2. General formalism

Previously, on the basis of the three-body formalism of the effective interaction of a charged particle and a bound complex[16−19], we have derived an expression for the polarization potential of the two-hadron bound complex that consists of charged and neutral hadrons (for example, the deuteron)[17,20,21], starting immediately from the Faddeev integral equations[22]. In the case that the interaction between the proton and the neutron composing the deuteron is central and S-wave, the electric dipole

polarizability of the deuteron is given by[15]

$$\alpha_E(^2\text{H}) = \frac{2}{3}\frac{e_p{}^2}{\hbar^2 c^2}\left(\frac{m_n}{m_{pn}}\right)^2\int_0^\infty \frac{dk k^2}{2\pi^2}\frac{\mid \psi_d'(k)\mid^2}{\frac{k^2}{2\mu_{pn}}+B_d}, \tag{3}$$

where e_p is the charge of the proton, μ_{pn} is the proton-neutron reduced mass, $\mu_{pn} = m_p m_n/m_{pn}$, $m_{pn} = m_p + m_n$ (m_p and m_n are the proton and neutron masses), $\psi_d'(k) \equiv d\psi_d(k)/dk$ is the first derivative of the deuteron wave function in the momentum space in the variable of the relative momentum k, and B_d is the binding energy of the deuteron. The formula (3) is in agreement with the expressions for α_E obtained in the case of the separable S-wave pair potential in Refs. 17, 20 and 21.

According to the expression for the electric polarizability of the two-particle bound complex (3), obtained assuming that the interaction in the P-wave orbital state is absent, the quantity α_E is completely determined by the wave function of the bound state of the complex. Hence, for the different interaction potentials producing identical bound-state wave functions (as an example, in the case of the two-body problem with the S-wave local Hulthén interaction potential and with the S-wave separable potential having the Yukawa formfactor), even if distinct the corresponding scattering wave functions, the expressions for the polarizability α_E should be the same.

In particular, from this it follows that a closer determination of α_E is anticipated in the event of fitting the potential parameters to bound-state data rather than scattering-state ones corresponding to higher energies.

Here we obtain the expression for the electric dipole polarizability of the three-hadron bound complex in the case when the system can form, apart from the continuum, only one bound state. We start from the general expression for the polarizability of the complex , considering the low-energy scattering of the three-body bound complex (with the binding energy B_0) by the field of a particle 1 having the electric charge e_1. For simplicity sake assume that the complex consists of one charged particle 2 and two neutral particles (3 and 4). The initial kinetic energy of the relative motion of the particle 1 and complex is taken to be far lower than the breakup threshold energy of the complex. The effective potential of the interaction between the charged particle 1 and complex was found within the framework of the rigorous Watson-Feshbach formalism[23,24] (for more details concerning application of the above formalism for the three-body system see Refs. 17 - 19, 21). The electric dipole polarizability α_E was determined as the strength of the polarization potential at asymptotically large distances between the particle 1 and complex, ρ_1, greatly exceeding the size of the complex,

$$V_{pol}(\rho_1) = -\alpha_E\frac{e_1^2}{2\rho_1^4}\ , \tag{4}$$

where α_E is given as

$$\alpha_E = -2 < \Psi_0 \mid (\mathbf{D}_2 \cdot \hat{\boldsymbol{\rho}}_1)G^Q(-B_0)(\mathbf{D}_2 \cdot \hat{\boldsymbol{\rho}}_1) \mid \Psi_0 >, \tag{5}$$

Here Ψ_0, $\mathbf{D}_2 = e_2\mathbf{r}_2$, and $G^Q(-B_0)$ are the wave function of the ground bound state of the three-hadron complex corresponding to the binding energy B_0 (normalized

to unit, $< \Psi_0 \mid \Psi_0 >= 1$), the operator of the dipole moment of the charged particle 2 having the charge e_2, and the truncated Green's operator of the complex $G^Q(E) = QG(E)$ at the energy $E = -B_0$, respectively. The full Green's operator of the complex $G(E)$ is given by $G(E) = (E - H_0 - V)^{-1}$, where H_0 is the kinetic energy operator and V is the total interaction potential, $V = v_{23} + v_{24} + v_{34}$, v_{ij} is the potential of the pair interaction between the particles i and j (the potentials are assumed to be energy-independent), $Q = 1 - P$, P is the projection operator onto the complex ground state, $P = \mid \Psi_0 >< \Psi_0 \mid$. The quantity $\boldsymbol{\rho}_1$ in Eqs. (4) and (5) is the radius vector specifying the relative position of the centre of mass of the complex (composed of the particles 2, 3 and 4) with respect to the charged particle 1, $\hat{\boldsymbol{\rho}}_1 \equiv \boldsymbol{\rho}_1/\rho_1$ is the unit vector along $\boldsymbol{\rho}_1$, giving the direction of the external electric field, created by the particle 1, $\mathbf{r}_2$ is the radius vector of the charged constituent of the complex, the particle 2, relative to the centre of mass of the complex.

Expressing the full Green's operator $G(E) = (E - H_0 - V)^{-1}$ through the transition matrix $T(E)$,

$$G(E) = G_0(E) + G_0(E)T(E)G_0(E), \tag{6}$$

we write the operator $G^Q(E)$, which appears in the expression (5) for the electric dipole polarizability of the three-body bound complex in the form

$$G^Q(E) = (1 - P)[G_0(E) + G_0(E)T(E)G_0(E)] , \tag{7}$$

where $G_0(E) = (E - H_0)^{-1}$ is the free Green's operator. The transition matrix $T(E)$ is defined by the Lippmann-Schwinger integral equation

$$T(E) = V + VG_0(E)T(E) . \tag{8}$$

In the case under consideration that the complex has only one bound state with the energy $E = -B_0$, the transition matrix may be written as the sum of the pole, $\check{T}(E)$, and smooth, $\tilde{T}(E)$, parts,

$$T(E) = \check{T}(E) + \tilde{T}(E) , \tag{9}$$

where

$$\check{T}(E) = \frac{\mid \Gamma_0 \rangle \langle \Gamma_0 \mid}{E + B_0} , \tag{10}$$

$$\mid \Gamma_0 \rangle = G_0^{-1}(-B_0) \mid \Psi_0 \rangle = V \mid \Psi_0 \rangle . \tag{11}$$

At the point $E = -B_0$ the operator $\tilde{T}(E)$ may be shown to have the form

$$\tilde{T}(-B_0) = -c_1 \mid \Gamma_0 \rangle \langle \Gamma_0 \mid + \cdots , \tag{12}$$

with

$$c_1 = \langle \Psi_0 \mid G_0(-B_0) \mid \Psi_0 \rangle . \tag{13}$$

The factored term in (12) is the smooth part of the dominant partial component of the three-body transition matrix $T(E)$ with zero relative orbital momenta. The additional terms in (12) indicated by the ellipsis contain the higher orbital components of $T(E)$ at $E = -B_0$, which are wholly smooth functions of E, however, being less important, they are disregarded here.

In view of (7) and (9), the operator $G^Q(E)$ may be written as

$$G^Q(E) = G_0^Q(E) + \check{G}^Q(E) + \tilde{G}^Q(E) \tag{14}$$

while the corresponding terms are given by

$$
\begin{aligned}
G_0^Q(E) &= (1-P)G_0(E), \\
\check{G}^Q(E) &= (1-P)G_0(E)\check{T}(E)G_0(E), \\
\tilde{G}^Q(E) &= (1-P)G_0(E)\tilde{T}(E)G_0(E).
\end{aligned}
\tag{15}
$$

The value of the matrix element

$$
\begin{aligned}
\langle \Psi_0 \mid (\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1)G^Q(E)(\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1) \mid \Psi_0 \rangle &= \langle \Psi_0 \mid (\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1)G_0^Q(E)(\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1) \mid \Psi_0 \rangle \\
&+ \langle \Psi_0 \mid (\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1)\check{G}^Q(E)(\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1) \mid \Psi_0 \rangle \\
&+ \langle \Psi_0 \mid (\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1)\tilde{G}^Q(E)(\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1) \mid \Psi_0 \rangle
\end{aligned}
\tag{16}
$$

appearing in the expression (5) at the energy $E = -B_0$ is obtained with the use of the relations (10)—(12) in the second and third terms on the right-hand side of Eq. (16) and with evaluating the indeterminate form of the type $0/0$ in the second term. Furthermore, it is easy to verify that the third term vanishes at the point $E = -B_0$.

As a result, the expression for the electric dipole polarizability (5) becomes then

$$
\begin{aligned}
\alpha_E = -\,&2\{\langle \Psi_0 \mid (\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1)G_0(-B_0)(\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1) \mid \Psi_0 \rangle \\
&+ \langle \Psi_0 \mid (\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1) \mid \Psi_0 \rangle \cdot [c_1 \langle \Psi_0 \mid (\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1) \mid \Psi_0 \rangle \\
&- \langle \Psi_0 \mid (\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1)G_0(-B_0) \mid \Psi_0 \rangle - \langle \Psi_0 \mid G_0(-B_0)(\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1) \mid \Psi_0 \rangle]\}.
\end{aligned}
\tag{17}
$$

Further simplification of the expression (17) for α_E occurs if the wave function of the bound complex Ψ_0 is characterized by a definite parity (as a consequence of the spatial reflection invariance of the interaction potential V). In such a case the matrix elements $\langle \Psi_0 \mid (\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1) \mid \Psi_0 \rangle$, $\langle \Psi_0 \mid G_0(-B_0)(\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1) \mid \Psi_0 \rangle$ and $\langle \Psi_0 \mid (\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1)G_0(-B_0) \mid \Psi_0 \rangle$ in (17) vanish due to integration over the angular variables. The expression (17) then reduces to the simple form

$$\alpha_E = -2\langle \Psi_0 \mid (\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1)G_0(-B_0)(\mathbf{D_2} \cdot \hat{\boldsymbol{\rho}}_1) \mid \Psi_0 \rangle \tag{18}$$

containing only the free Green's operator at the negative energy $E = -B_0$.

The bound complex being considered below is a three-hadron nucleus composed of the proton p (the charged particle 2), the neutron n (the neutral particle 3) and the neutral hadron h (the particle 4). The symbol h stands for n (the neutron) in the case of the triton and for Λ (the lambda hyperon) in the case of the lambda hypertriton.

In the momentum representation the operators of the dipole moment $\mathbf{D}_2$ and the free propagator $G_0(-B_0)$ in the Eq. (18) are given by

$$\mathbf{D}_2 = ie_p\left(\tfrac{m_n}{m_{pn}}\nabla_\mathbf{k} + \tfrac{m_h}{m_{pnh}}\nabla_\mathbf{p}\right),$$

$$\langle \mathbf{kp} \mid G_0(-B_0) \mid \mathbf{k'p'}\rangle = -(2\pi)^6\delta(\mathbf{k}-\mathbf{k'})\delta(\mathbf{p}-\mathbf{p'})\left(B_0 + \tfrac{k^2}{2\mu_{pn}} + \tfrac{p^2}{2\mu_{pn,h}}\right)^{-1}. \tag{19}$$

where $\mathbf{k}$ and $\mathbf{p}$ are the Jacobi momentum variables describing the relative motion of particles p and n and that of the particle h with respect to the centre of mass of (p,n),

$$\mathbf{k} = \frac{m_n\mathbf{k}_p - m_p\mathbf{k}_n}{m_{pn}}, \quad \mathbf{p} = \frac{m_h(\mathbf{k}_p + \mathbf{k}_n) - m_{pn}\mathbf{k}_h}{m_{pnh}}. \tag{20}$$

Here $\mathbf{k}_i$ is the momentum of the particle i, $\mu_{pn,h}$ is the reduced mass of the system (p, n) with the mass $m_{pn} = m_p + m_n$ and the hyperon h with the mass m_h, $\mu_{pn,h} = m_{pn}m_h/m_{pnh}$, $m_{pnh} = m_p + m_n + m_h$, the binding energy of the three-hadron bound complex B_0 is equal to the sum of the deuteron binding energy $B_d = \kappa_d^2/2\mu_{pn}$ and the separation energy of the hyperon h, $B_h = \kappa_h^2/2\mu_{d,h}$, $B_0 = B_d + B_h$, $\mu_{d,h} = m_d m_h/m_{dh}$, m_d is the deuteron mass, $m_{dh} = m_d + m_h$, $\Psi_0(\mathbf{k}, \mathbf{p})$ is the normalized wave function of the three-hadron nucleus in momentum space, $\nabla_{\mathbf{k}} \equiv \frac{\partial}{\partial \mathbf{k}}$ is the gradient operator.

When the full wave function Ψ_0 in the general formula (18) has the form of the product of spatial and spin functions, we may omit spin variables from consideration. In the momentum space, substituting the expressions (19) into (18), the formula for the electric dipole polarizability of the three-hadron bound system may be written as

$$\alpha_E(pnh) = 2\frac{e_p^2}{\hbar^2 c^2} \int \frac{d\mathbf{k}d\mathbf{p}}{(2\pi)^6} \frac{|\langle \mathbf{kp}|\, \hat{\boldsymbol{\rho}}_1 \cdot \left(\frac{m_n}{m_{pn}}\nabla_{\mathbf{k}} + \frac{m_h}{m_{pnh}}\nabla_{\mathbf{p}}\right)\Psi_{pnh}\rangle|^2}{\frac{k^2}{2\mu_{pn}} + \frac{p^2}{2\mu_{pn,h}} + B_0}, \qquad (21)$$

where Ψ_{pnh} is the spatial wave function of the three-hadron nucleus.

We are reminded that the formula for the electric polarizability of the three-hadron bound system (21) has been derived assuming that the higher orbital components of the off-shell three-body transition matrix at the negative energy $E = -B_0$ are negligibly small. Similar to the expression (3) for the electric dipole polarizability of the two-body complex, the obtained expression for the polarizability of the three-body complex is essentially determined by the first derivatives of its bound-state wave function with respect to the Jacobi momentum variables.

Below, using a simple model wave function we employ our formula (21) to evaluate the electric dipole polarizability of the simplest three-body nucleus, the triton ^{3}H containing only one charged particle (the proton).

It is worth noting that the parametrized analytical form for the Faddeev triton wave function generated with the Reid soft-core potential in Ref. 25 is quite suitable to use in the expression of the type (21) with the aim to calculate the triton electric polarizability of the triton.

3. Model wave function of the triton

In this article we study the electric dipole polarizability of the triton nucleus containing the proton (the particle 2) and two neutrons (the particles 3 and 4). To estimate the polarizability, we use the clustered (deuteron + neutron) triton wave function that must be antisymmetrized with respect to the identical fermions.

For simplicity sake, without introducing the isobaric formalism, we have to antisymmetrize only with respect to the two neutrons. This antisymmetrization naturally occurs, if we shall restrict our consideration only the dominant symmetric part of the space wave function of the triton, since its spin wave function $\chi_t \equiv \chi_{SM_S}$ (with $S = 1/2$) is already antisymmetric relative to the permutation of the neutrons,

$$\chi_{\frac{1}{2}\frac{1}{2}}(3, 4; 2) = \frac{1}{\sqrt{2}}\left(\alpha_3\beta_4 - \alpha_4\beta_3\right)\alpha_2. \qquad (22)$$

where α_i and β_i are the spin functions of the particle i with the spin projections $+1/2$ and $-1/2$.

Thus, in the framework of the cluster model of the triton as a $n + d$ system, the space part of the triton wave function must be symmetric in the interchange of the two neutrons,

$$\Psi_t(\mathbf{k}, \mathbf{p}) = \frac{N_n}{\sqrt{2}} \left[\psi_d(k_{23})\phi_n(p_4) + \psi_d(k_{24})\phi_n(p_3) \right], \tag{23}$$

where $\phi_n(p)$ is the normalized bound-state wave function describing relative motion of the neutron and centre of mass of the deuteron, the Jacobi momentum variables $\mathbf{k}_{23} \equiv \mathbf{k}$ and $\mathbf{p}_4 \equiv \mathbf{p}$ are defined by the relations (20),

$$\mathbf{k}_{24} = \frac{m_n}{m_{pn}}\mathbf{k} + \frac{m_p m_{pnn}}{m_{pn}{}^2}\mathbf{p}, \quad \mathbf{p}_3 = \mathbf{k} - \frac{m_n}{m_{pn}}\mathbf{p}, \tag{24}$$

and the coefficient N_n ensures normalization of the whole triton wave function to 1,

$$N_n = \left(1 + \langle \psi_d(k)\phi_n(p) + \psi_d(k_{24})\phi_n(p_3) \rangle \right)^{-1/2}.$$

Substituting the model wave function (23) into the formula (21), we get the following expressions for the electric dipole polarizabilities of the triton,

$$\alpha_E(^3\text{H}) = \alpha_E^0(pnn) + \alpha_E^{ex}(pnn) \tag{25}$$

with

$$\alpha_E^0(pnn) = \frac{4}{3}\frac{e_p{}^2}{\hbar^2}\frac{m_p m_n}{m_{pn}}N_n^2 \int_0^\infty \frac{p^2 dp}{2\pi^2} \left\{ \left(\frac{m_n}{m_{pnn}}\right)^2 I_n(p) \left[\frac{d\phi_n(p)}{dp}\right]^2 \right.$$
$$\left. + \left(\frac{m_n}{m_{pn}}\right)^2 J_n(p) \left[\phi_n(p)\right]^2 \right\} \tag{26}$$

and

$$\alpha_E^{ex}(pnn) = 4\frac{e_p{}^2}{\hbar^2}\frac{m_p m_n}{m_{pn}}N_n^2 \int\int \frac{d\mathbf{k}d\mathbf{p}}{(2\pi)^6}\frac{1}{k^2 + [C_n(p)]^2}$$
$$\cdot \left\{ \left(\frac{m_n}{m_{pnn}}\right)^2 \psi_d(k)\psi_d(k_{24})\frac{d\phi_n(p)}{dp}\frac{d\phi_n(p_3)}{dp_3}(\hat{\boldsymbol{\rho}}_1 \cdot \hat{p})(\hat{\boldsymbol{\rho}}_1 \cdot \hat{p}_3) \right.$$
$$\left. + \left(\frac{m_n}{m_{pn}}\right)^2 \frac{d\psi_d(k)}{dk}\frac{d\psi_d(k_{24})}{dk_{24}}\phi_n(p)\phi_n(p_3)(\hat{\boldsymbol{\rho}}_1 \cdot \hat{k})(\hat{\boldsymbol{\rho}}_1 \cdot \hat{k}_{24}) \right\}. \tag{27}$$

Here the hat signifies a unit vector, the functions $I_n(p)$ and $J_n(p)$ are defined by

$$I_n(p) = \int_0^\infty \frac{dk k^2}{2\pi^2}\frac{[\psi_d(k)]^2}{k^2 + [C_n(p)]^2}, \quad J_n(p) = \int_0^\infty \frac{dk k^2}{2\pi^2}\frac{[d\psi_d(k)/dk]^2}{k^2 + [C_n(p)]^2}, \tag{28}$$

$$[C_n(p)]^2 \equiv \frac{m_p m_n}{m_{pn}m_n}\left\{\frac{m_{pnh}}{m_{pn}}p^2 + \frac{m_{dh}}{m_d}\kappa_h^2\right\} + \kappa_d^2.$$

The wave functions $\psi_d(k)$ and $\phi_n(p)$ in the equations (26)–(28), which determine the model wave function of the triton, were found by solving analytically the

corresponding two-body problems. Note that the $p - n$ interaction is relatively weak resulting to rather small deuteron binding energy. This interaction can be well described using even a simple separable rank-1 potential. Another situation exists in the case of the $n - d$ interaction that is stronger than the $p - n$ interaction. In addition to the $n - d$ bound ground state (the triton), the $n - d$ interaction supports also one virtual state (that is reflected in a rather small value of the $n - d$ scattering length). In this connection the $n - d$ interaction can be described by a separable potential of the rank no less than 2 or by a local potential.

Here, in the case of the triton, both the wave functions $\psi_d(k)$ and $\phi_n(p)$ were determined by solving the corresponding two-body bound-state problems with the local Hulthén (H) potential[26],

$$v^H(r) = -v_0 \left[\exp(qr) - 1\right]^{-1} , \qquad (29)$$

employed to describe the proton-neutron interaction, $v_{pn}^H(r)$, and the effective interaction between the neutron and the deuteron (as a structureless object), $v_n^H(\rho)$. (The radius vector variables $\mathbf{r}$ and $\boldsymbol{\rho}$ in configuration space correspond to the variables $\mathbf{k}$ and $\mathbf{p}$ in momentum space.)

4. Calculations and discussion of results

The electric dipole polarizability of the triton $\alpha_E(^3\text{H})$ was calculated by the formulae $(25) - (28)$.

The parameters of the $p-n$ interaction potentials were fitted to the experimental values of the deuteron binding energy B_d and the triplet $p-n$ scattering length $^3a_{pn}$,

$$B_d = 2.224575(9) \text{ MeV (Ref. 27)}, \quad ^3a_{pn} = 5.424(3) \text{ fm (Ref. 28)}. \qquad (30)$$

The parameters of the effective $n - d$ interaction potential (29) were determined using the experimental values of the separation energy of the neutron (needed to remove one neutron from the triton, $B_n = B_t - B_d$) and the doublet $n - d$ scattering length $^2a_{nd}$,

$$B_t = 8.481855(13) \text{ MeV (Ref. 29)}, \quad ^2a_{nd} = 0.65(4) \text{ fm (Ref. 30)}. \qquad (31)$$

The fitted values of the parameters ($\gamma = v_0 q^{-3}$ and $\beta = q + \kappa$) of the local Hulthén $p - n$ and $n - d$ ineraction potentials (29), v_{pn} and v_n, were found to be

$$2\mu_{pn}\gamma_{pn} = 1.3184 \text{ fm}, \quad \beta_{pn} = 1.3146 \text{ fm}^{-1} ; 2\mu_{nd}\gamma_n = 5.4689 \text{ fm}, \quad \beta_n = 0.9552 \text{ fm}^{-1}. \qquad (32)$$

For the deuteron, the electric dipole polarizability obtained from Eq.(3) for the local Hulthén potential with the parameters (32) takes the value $\alpha_E^H(^2\text{H}) = 0.6442 \text{ fm}^3$. It is known [7,31] that the S-wave asymptotic normalization constant $A_S(^2\text{H})$ accounts for most of the polarizability $\alpha_E(^2\text{H})$ having regard to a rather high probability of that the slightly bound nucleons in the deuteron are at distances outside of the range of the nuclear force. If both the parameters of the potential are fitted to only the data that concerns to the bound $p - n$ state, for example, with the use of the binding energy B_d (30) and

$$A_S(^2\text{H}) = 0.8845(8) \text{ fm}^{-1/2} \text{ (Ref. 32)}, \qquad (33)$$

we find for the deuteron polarizability the value

$$\alpha_E^{\mathrm{H}}(^2\mathrm{H}) = 0.6292 \text{ fm}^3 . \tag{34}$$

With the use of the tensor separable potential that corresponds to $A_S = 0.8843$ fm$^{-1/2}$ the deuteron polarizability increases still further — to the value $\alpha_E = 0.6311$ fm^3 (Ref. 8) — approaching the values obtained with the realistic potentials, $\alpha_E = 0.6328(17)$ fm^3 (Ref. 7).

In this paper the electric dipole polarizability of the triton given by the formulae (25) – (28) has been calculated with the use of the local Hulthén potential that allows of finding the wave functions $\psi_d(k)$ and $\phi_n(p)$ in the analytical form. Fitting the parameters of the potential to the data for B_d and $^3a_{pn}$ of the $p - n$ system (30) and for B_n and $^2a_{nd}$ of the $n - d$ system, (30) and (31), we have obtained for the triton polarizability the value

$$\alpha_E^{\mathrm{H}}(^3\mathrm{H}) = 0.235 \text{ fm}^3 . \tag{35}$$

In this case, the direct and exchange terms in (25) are found to contribute to the triton polarizability almost equally:

$$\alpha_E^0(pnn) = 0.109 \text{ fm}^3 , \quad \alpha_E^{ex}(pnn) = 0.126 \text{ fm}^3 .$$

If the parameters of the $p - n$ interaction potential are fitted to the data characterizing only $p - n$ bound state (the deuteron), B_d and $A_S(^2\mathrm{H})$ (the S-wave asymptotic normalization), we obtain a slightly less value of the triton polarizability, $\alpha_E^{\mathrm{H}}(^3\mathrm{H}) = 0.225$ fm^3 . The decrease of the polarizability in this case occurs due to the decrease of the quantity A_S (the experimental value of A_S (33) is less than the value of A_S for the Hulthén wave function corresponding to the potential fitted to the data B_d and $^3a_{pn}$ ($A_S(^2\mathrm{H}) = 0.8960$ fm$^{-1/2}$)).

Unfortunately, the value $\alpha_E(^3\mathrm{H})$ has not been measured yet. Although our tentative estimate of the triton polarizability, carried out on the basis of the direct calculation by the formula (21) but using the cluster wave function, leads to the result for $\alpha_E(^3\mathrm{H})$ that is consistent with one of two data for the polarizability of the mirror nucleus $^3\mathrm{He}$ (Ref. 1), a new calculation of $\alpha_E(^3\mathrm{H})$ with the use of a more reasonable triton wave function in (21) would be valuable.

5. Summary and conclusions

Leaning upon on the analytical structure of the three-body transition matrix, a consistent formalism for determination of the electric dipole polarizability of a three-hadron bound complex consisting of one charged and two neutral particles and having only one stable bound state has been worked out. A simple expression for the electric dipole polarizability of the three-hadron bound system has been derived assuming that the higher orbital components of the three-body off-shell transition matrix at the negative energy $E = -B_0$ are negligibly small. In this case, the polarizability is expressed in terms of the first partial derivatives of the bound-state wave function with respect to the Jacobi momentum variables of the complex (Eq. (21)).

Applying the cluster model for the triton wave function and the Hulthén interaction potential to describe the $p - n$ and $n - d$ bound systems, we have obtained for the electric polarizability of the triton the approximate estimate $\alpha_E^{\mathrm{H}}(^3\mathrm{H}) = 0.23$ fm^3(using the low energy data for B_d and $^3a_{pn}$ of the $p - n$ system and for B_n and $^2a_{nd}$ of the $n - d$ system). Under conditions that there is presently no direct measurement of the quantity $\alpha_E(^3\mathrm{H})$ and the results of the experiments for the polarizability of the mirror nucleus $^3\mathrm{He}$ (Refs. 3 and 4), are inconsistent, more complicated calculations of the triton polarizability on the basis of the expression (21) and modern wave functions for the $^3\mathrm{H}$ bound state are worth to be performed.

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